

2.2

Power Functions with Modeling

Definition: Any function that can be written in the form

$f(x) = k \bullet x^a$, where k and a are nonzero constants, is a **power function**.

The constant a is the **power**. The k is the **constant of variation** or **constant of proportion**. We say $f(x)$ **varies as** the a^{th} power of x , or $f(x)$ is **proportional to** the a^{th} power of x .

Example: $A = \pi r^2$ the power is 2 and the constant of variation is π

Power function formulas with **positive powers** are statements of **direct variation**.

Power function formulas with **negative powers** are statements of **inverse variation**.

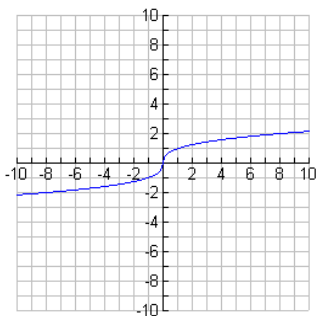
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Analyzing a Power Function:

Ex. State the power and constant of variation for the function, graph it, and analyze it.

$$f(x) = \sqrt[3]{x}$$

Because $f(x) = \sqrt[3]{x} = x^{\frac{1}{3}} = 1 \bullet x^{\frac{1}{3}}$, its power is $1/3$, and its constant of variation is 1.



Domain: All reals

Range: All reals

Continuous

Increasing for all x

Symmetric with respect to the origin (odd)

Not bounded above or below

No local extrema

No asymptotes

End Behavior: $\lim_{x \rightarrow -\infty} \sqrt[3]{x} = -\infty$ and $\lim_{x \rightarrow \infty} \sqrt[3]{x} = \infty$

Note: This is the inverse of x^3 .

Definition of a monomial function – any function that can be written as $f(x) = k$ or $f(x) = k \cdot x^n$ where k is a constant and n is a **POSITIVE INTEGER**.

The Cubing Function:

$$f(x) = x^3$$

Domain: All reals

Range: All reals

Continuous

Increasing for all x

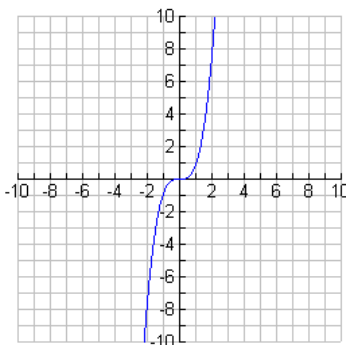
Symmetric with respect to the origin (odd)

Not bounded above or below

No local extrema

No asymptotes

End Behavior: $\lim_{x \rightarrow -\infty} x^3 = -\infty$ and $\lim_{x \rightarrow \infty} x^3 = \infty$



See pg. 192

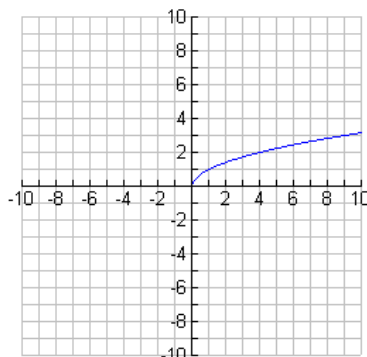
The Square Root Function:

$$f(x) = \sqrt{x}$$

Domain: $[0, \infty)$

Range: $[0, \infty)$

Continuous on $[0, \infty)$



Increasing on $[0, \infty)$

No symmetry

Bounded below but not above

Local minimum at $x = 0$

No asymptotes

End Behavior: $\lim_{x \rightarrow \infty} \sqrt{x} = \infty$